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A booklet designed for elementary school students to be used independently from and in addition to the regular classroom curriculum in mathematics is presented. The purpose is to teach the fifth- or sixth-grade student to use good judgment and reasoning in locating the decimal point. A discussion of the purpose and importance of being able, quickly and accurately, to estimate values introduces the student to the topic. Sample problems, answers, and examples are presented. An example of the mental method used in this booklet is: ".24 is almost ¼. One-fourth x 4.1 is about 1 ; .984 is about 1. The point is placed correctly." The teacher should identify potential users, orient them to the area to be explored, and stimulate and maintain their interest in sustained independent effort.

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INDEPENDENT LEARNING BOOKLET

ESTIMATION

THE DECIMAL POINT IN DIVISION

DEPARTMENT OF EDUCATION • SAN DIEGO COUNTY

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U. S. DEPARTMENT OF HEALTH, EDUCATION AND WELFARE
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TO THE TEACHER

This Independent Learning booklet is designed to help you open new avenues of learning for students who show a special interest in mathematics and a readiness to study on their own. Such students will be able to work directly from printed materials, using abstract ideas to follow a line of mathematical reasoning. Occasionally, of course, even the ablest of them will need your help to get over some of the rough spots, but your main concern in connection with the booklet will be to identify students who can benefit from using it, to orient them to the area to be explored, and to stimulate and maintain their interest in sustained independent effort.

Since the topic of the booklet trends outward from the regular curriculum, the challenge to do this independent work should arise from students' own searching in the field or from new interests that you have succeeded in awakening. Students will profit most if they elect voluntarily to do the work rather than receiving it as an assignment or as a means of earning extra credit.

The material is so organized that students may progress at their own best pace. They should be encouraged to move ahead in discovering for themselves meanings and approaches in mathematics beyond those usually taught in the classroom, but there should be no requirement to complete parts or all of the booklet, or to follow a set schedule.

While serious effort will be necessary, students should find the work enjoyable because of their interest in the subject. They should gain a real sense of satisfaction through working individually or with other students in independently achieving understanding, overcoming temporary difficulties, and evaluating their own progress. Your encouragement and occasional direct help will of course be needed.

This booklet obviously is not aimed at developing complete understanding of an area of mathematics. Its purpose is rather to provide an introduction, or an approach novel to the student, which will create new interests and the desire to go on from here to further study in other sources.

A brief teacher's guide has been prepared to assist you in using this booklet with your students. Copies of the guide are distributed to your district office by the Department of Education, San Diego County. Additional copies are always available.

This Independent Learning Booklet is part of an experimental project to provide mathematics materials for able pupils and to develop a mathematics program adapted to individual differences. The project is based on several studies and exploratory programs carried on in San Diego County during the past several years, such as those sponsored by the San Diego Metropolitan Education Study Council and the Assistant Superintendents Group. Objectives of this phase of the project are to stimulate and widen pupil interest in various areas of mathematics; to develop deeper insights into mathematical principles, concepts, and relationships; to introduce aspects of modern mathematical thinking; and to develop motivation for able pupils to continue to study and explore mathematics and related fields.

The project is coordinated by the Department of Education, San Diego County, and directed by a steering committee selected from members of the San Diego Metropolitan Education Study Council. Support for the project is derived from funds provided under Title III of the National Defense Education Act of 1958 and matching funds from the participating school districts and the San Diego County Department of Education.

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Other Independent Learning Booklets have been prepared and will be announced as soon as they are ready for distribution.

INDEPENDENT LEARNING BOOKLET

Estimation: The Decimal Point in Division

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INTRODUCTION

The purpose of this booklet is to show you how to use good judgment and reasoning in locating the decimal point in division. If you like arithmetic you will find the methods introduced here not only efficient but fun to do.

You will be learning directly from this booklet, so follow the explanations carefully. Your teacher will help you if you really get "stuck," but don't ask for help until you have worked through a section several times. You will feel more pleased with your accomplishments if you have achieved them on your own.

A word of advice: The booklet is divided into several parts. You will have smooth sailing if you do enough practice with the ideas developed in each part to be skillful in using them before you go on to the next part.

While you should have pencil and paper ready to use when you need them, do as much of the work as possible without writing. Since other boys and girls will be using this same material, please do not write anything in the booklet itself.

Recalling Some Ideas

To get off to a good start, let's first review some vocabulary and ideas with which you are already acquainted. Having them clearly in mind is important to your success with the new ways of working introduced in this booklet.

Digits and Numerals

The symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are called **digits.** We use these ten digits in writing all the **numerals** in the decimal place-value system of notation. Just as words stand for ideas, so numerals stand for number ideas. The numeral **340** stands for the number idea we call "three hundred forty." The digits of 340 (3, 4, and 0) could be used to write different numerals, such as 304, 430, 403, etc., each of which is a distinct number idea.

When a digit stands alone, for example a 2, a 5, or an 8, we may speak of it as a digit or as a numeral.

Integers

Integers are whole numbers rather than fractional numbers. The following numerals stand for integers:

6 53 108 1790

The following numerals stand for fractions:

$\frac{1}{2}$ $\frac{5}{8}$ .1 .25

Sometimes we speak of the part of a number that is an integer. The 9 of 9.1 and the 8 of 8.6 are examples.

Subtraction

Subtraction is the inverse, that is, the opposite or undoing, of addition. You know that when you add, you **combine** quantities. You can see the relationship of addition to subtraction in this illustration: $2 + 3 = 5$ and $3 + 2 = 5$, therefore $5 - 3 = 2$ and $5 - 2 = 3$.

Minuend

The **minuend** in subtraction is the quantity from which another quantity is to be subtracted. For example, 5 is the minuend in the subtractions above.

Subtrahend

The **subtrahend** is the quantity which is to be subtracted from another quantity. For example, in $5 - 2$ the subtrahend is 2.

Difference or Remainder

The result of subtracting is called the **difference** or **remainder.**

For example:

$$\begin{array}{r} 5 \\ -2 \\ \hline 3 \end{array}$$

5 Minuend
−2 Subtrahend
3 Difference or Remainder

Minuend − Subtrahend = Difference or Remainder

Division

Division is the inverse of multiplication. You know that when you multiply, you combine equal quantities to find a total, called the product. For example, you combine 4 threes to get the product 12. Since 4 threes equal 12, you can take 4 threes out of 12. We say that $4 \times 3 = 12$, therefore $12 \div 3 = 4$.

Dividend — The **dividend** is the quantity which is to be divided.

Divisor — The **divisor** is the quantity by which the dividend is divided.

Quotient — The **quotient** is the quantity resulting from the division.

For example:

$$3\overline{)\,12}\quad\text{and}\quad 12 \div 3 = 4$$

Dividend	12
Divisor	3
Quotient	4

$$\text{Divisor}\overline{)\,\text{Dividend}}\qquad \text{Quotient}$$

Dividend ÷ Divisor = Quotient

$8\overline{)\,272}$

Steps in Division—Follow the steps with the example at the left to see how the division process can be performed. Because of the way the decimal place-value system works, we can begin at the left and work toward the right. We divide the larger place values first and work with part of the dividend at a time. We call this a **partial dividend.**

Step 1

$8\overline{)\,272}$

1. Select the first partial dividend. Is 8 contained in 2 at least once? No. In 27? Yes, therefore 27 is our first partial dividend.

Step 2

$$8\overline{)\,272}\qquad 3$$

2. Divide 27 by 8 to get the first quotient digit. How many 8's in 27? $3 \times 8 = 24$, $4 \times 8 = 32$; 3 must be correct. Write the 3 in tens' place because the 27 stands for 27 tens.

Step 3

$$8\overline{)\,272}\qquad 3$$
$$24$$

3. Multiply and compare. $3 \times 8 = 24$. Is 24 no larger than 27?

Step 4

```
      3
   ┌─────
8 ) 2 7 2
    2 4
    ---
      3
```

4. Subtract 24 from 27. Compare to see that the difference (3) is smaller than the divisor (8).

Step 5

```
      3
   ┌─────
8 ) 2 7 2
    2 4
    ---
      3 2
```

5. Three tens and 2 ones remain to be divided. Bring down the 2 ones to make the next partial dividend, 32.

```
      3 4
   ┌─────
8 ) 2 7 2
    2 4
    ---
      3 2
      3 2
```

Now do these steps:
Divide 32 by 8.
Write the 4 in ones' place.
Multiply and compare.
Subtract and compare.
There is no more to be divided; the division is completed.

```
        2 5 1
   ┌─────────
3 2 ) 8 0 3 5
      6 4
      -----
      1 6 3
      1 6 0
      -----
          3 5
          3 2
          ---
            3
```

Now study this example, which has a two-digit divisor. The first partial dividend is 80. The 8 is in thousands' place and the 0 is in hundreds' place; there are 80 hundreds. Because of the way our decimal place-value system works, when we are performing the division we need only to think, How many 32's are there in 80? The next partial dividend is 163. The last partial dividend is 35 since we are not "carrying out" to decimal fractions. We have 3 left over, which we call a remainder. The quotient could be written $251\frac{3}{32}$.

In the example above the partial dividends 80, 163, and 35 are also minuends, while 64, 160, and 32 are subtrahends. We obtained these subtrahends by multiplying the divisor by the appropriate digit of the quotient:
$2\times32=64$, $5\times32=160$, and $1\times32=32$.

Checking Division—Division can be checked by multiplication.

Division	Multiplication
$\begin{array}{r} 251 \\ 32\overline{)8035} \\ 64 \\ \hline 163 \\ 160 \\ \hline 35 \\ 32 \\ \hline 3 \end{array}$	$\begin{array}{r} 32 \\ \times 251 \\ \hline 32 \\ 160 \\ 64 \\ \hline 8032 \\ + \quad 3 \\ \hline 8035 \end{array}$

When we divide we start with the largest place values and work from left to right. When we multiply we start with the smaller place values and work from right to left. (Other ways of working are possible; these are the ones ordinarily used.) Can you see the partial dividends of the division above in the partial products of the multiplication?

Estimating and Approximating

When we **estimate** in arithmetic we use our best judgment about an answer without working it out exactly, that is, we **approximate** the answer, or approach it closely. Estimating and approximating help us judge the reasonableness of the results of calculation.

Rounding

Rounding numbers is helpful in estimating; when we "round" we approximate the value of the number. For example, if we want to estimate the quotient of $32.13 \div 5.1$ we can think of 30 divided by 5 and estimate that the quotient will be close to 6 (the exact quotient is 6.3). You may want to study the material on rounding numbers in your textbook.

Do you have each point above clearly in mind? If so, you are ready to learn the new ideas which are introduced in this booklet. You can refer to this part of your booklet again if it will help you as you work with the following sections.

ESTIMATION: THE DECIMAL POINT IN DIVISION

You know that this is an age of such marvels of engineering as rockets. Are you aware that it is also an age of calculators and computers? Many of you will be using these machines some day. They can add, subtract, multiply, and divide any numbers you feed into them. While the devices will do the calculations for you faster than you can do them by yourself, it is up to you to know and understand the relationships both of the numbers you feed in and those which come out. The answers the devices give will "make sense" only if you are aware of the place values of the numerals which stand for these numbers.

A key skill in using many calculating machines is locating the decimal point because the decimal point shows where the units' (ones') place is in a numeral.

hundreds	tens	ones	tenths	hundredths
5	5	5 .	5	5

Five hundred fifty-five and fifty-five hundredths

The ones' place is the center of the decimal place-value system of notation. As we move to the left, each place value is multiplied by ten. As we move to the right, each place value is divided by ten. This relationship is shown in the chart at the left.

In the numeral **555.55** the center is 5 ones. The 5 at the left of ones' place stands for 5×10, or 50; the 5 at its right stands for $5 \div 10$, or .5, and so on.

You may already know how to use the slide rule. It is one of the calculating devices which call for the skill of locating the decimal point. On it you can work such an example as **.0108 ÷ .54** and get the quotient very rapidly without using pencil and paper. But you still have the problem of placing the decimal point in your answer.

Do you know how to locate the decimal point at a glance without using pencil and paper, or are you limited to using the caret method as shown in the example at the left? In the caret method we start with the divisor, .54 in this example, and count the number of decimal places at the right of the ones' place. We put a caret (∧) at this spot. (Although .54 has no ones' digit, the decimal point shows where the ones' place is.) Then we turn to the dividend, **.0108,** and starting with the digit at the right of ones' place, we count off the same number of places as we did in the divisor. We put a caret at this spot too. We place the decimal point in the quotient directly above the caret in the dividend.

```
      .02
.54∧).01∧08
      108
```

You can see that when we use the caret method we need to write the numerals in the form shown in our example, and this takes time. Also, it is a mechanical rather than a thoughtful way of working. Often when we use it we neglect to think about the reasonableness of the answers we obtain.

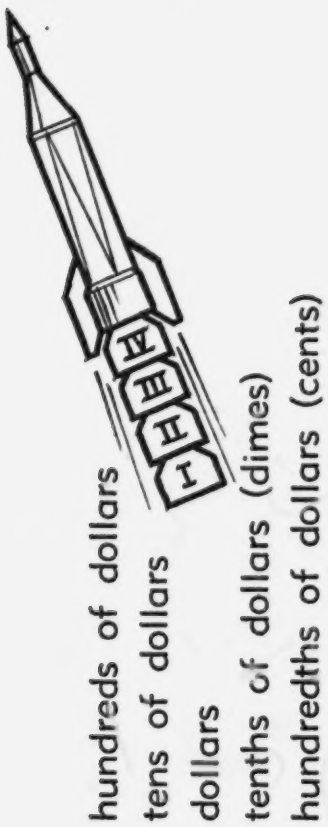
The method described in the following pages will not only help you learn to locate the decimal point in quotients mentally, without the necessity of writing the full example, but will also give you confidence in the **reasonableness** of the quotients you obtain. This method simply calls for one thing on your part: **good judgment,** or **reasoning.** You do not need to memorize a rule. You just use what you already know about arithmetic.

You can learn the method and reach the goal of quickly and confidently placing the decimal point by studying the following four parts of this booklet. We might liken what we will do to sending into orbit a rocket with four stages. Remember, a rocket will not go into orbit unless each stage operates properly in its turn. So be sure that you understand each explanation and can apply your learning before you go on to the next. If you do this, your rocket will achieve an orbit upon which you can depend—the orbit of real understanding.

You have just completed the plans for your rocket. As a countdown, reread the material on pages 3 through 7.

STAGE I

Dividing Dollars and Cents



	hundreds of dollars	tens of dollars	dollars	tenths of dollars (dimes)	hundredths of dollars (cents)
$	2	2	2	2	2

Two hundred twenty-two dollars and twenty-two cents.

Your rocket will complete its first stage if you gain a good understanding of division with dollars and cents in the dividend.

Suppose you want to divide eight dollars into four equal shares. $8.00 ÷ 4 = ? You can quickly see that **2 0 0** are the digits in the quotient. But should the quotient be two dollars, twenty dollars, or twenty cents? The answer obviously is two dollars. Then, where does the decimal point belong? Yes, at the right of the 2 to show that **$2.00** is the answer.

In numerals which stand for United States money, dollars are in the ones' place. The chart at the left illustrates some of the relationships in our monetary system.

Where should the decimal point be located in the quotient of this example: **$17.00 ÷ 4 = $425?*** Is one of four equal parts of seventeen dollars approximately four dollars, forty dollars, or four hundred dollars? You know that it is about four dollars. Then, where does the point belong? Yes, at the right of the 4 to give us **$4.25** as the quotient.

Suppose that you are dividing $24.08 into 8 equal shares. Where would you locate the decimal point in the quotient: **$24.08 ÷ 8 = $301**? Should the quotient be approximately three dollars, thirty dollars, or three hundred dollars? What about 30 cents? Is **$3.01** the correct quotient?

*In examples written with the equal sign, a temporary quotient is shown in color because the statement is not true until you have placed the decimal point correctly.

Where should the point be located in this quotient: $69.36 \div 3 = \$2312$?

You might think, "The quotient can't be as much as two thousand dollars or two hundred dollars. It can't be as little as two dollars. The decimal point must belong at the right of the 3; **$23.12** is my answer."

Now try the examples in SET A following. Copy each example. Use good judgment to place the point in each quotient. Then turn to page 28 to check your thinking.

SET A

	a	b
1.	$\$4.32 \div 4 = \$108$	$\$12.60 \div 6 = \$210$
2.	$\$64.96 \div 2 = \$3248$	$\$50.00 \div 5 = \$1000$
3.	$\$145.60 \div 7 = \$2080$	$\$345.36 \div 3 = \$11512$

Suppose that you want to work the following example: $\$.25 \div 5 = ?$ 5 is obviously the quotient digit. What does this 5 stand for? Does it represent 5 dollars? 5 cents? The answer, of course, is 5 cents. Then, where does the decimal point belong? You must **annex** a zero at the left, mustn't you, to place the point so that the 5 stands for 5 cents? **$.05** (To **annex** means to attach or affix.)

Where does the point go in this quotient: **$1.10÷5=$22?** You could think, "If I divide a little more than one dollar into 5 equal shares, there can't be as much as a dollar in each share. There would be about 20 cents in each share. The quotient must be 22 cents." **$.22**

Now copy the quotients of SET B. Use good judgment to place the decimal points correctly. Remember, you may have to annex zeros. Then check your thinking on page 28.

SET B

	a	b
1.	$\$.48 \div 4 = \$12$	$\$1.25 \div 5 = \$25$
2.	$\$1.33 \div 7 = \$19$	$\$.18 \div 3 = \$6$
3.	$\$4.28 \div 4 = \$107$	$\$9.30 \div 10 = \$93$
4.	$\$1152.09 \div 9 = \$12801$	$\$1.65 \div 15 = \$11$
5.	$\$6.00 \div 6 = \$100$	$\$30.00 \div 8 = \$375$
6	$\$621.30 \div 3 = \$20710$	$\$108 \div 12 = \$9$

STAGE II

Decimal Fractions in the Dividend

Now that you understand division with dollars and cents, the second stage of your rocket is about ready to fire.

You have seen that you can easily locate the decimal point in the quotient when the dividend is money. Now you will be working with decimal fractions which do not stand for money. Study the examples at the left.

a. $4\overline{)12} = ?$ You know that 3 is the quotient in **a**.

b. $4\overline{)12.0} = ?$ What is the quotient in **b**? Yes, 3.0 is correct.

c. $4\overline{)12.00} = ?$ What about **c**? 3.00 is the quotient.

Do the decimal points and annexed zeros in the dividends and quotients of **b** and **c** confuse you? They will not if you stop to think that in each case you are dividing only the whole number (or integer) **12** into **four** equal parts.

Now study this example: **2.0 ÷ 4 = ?** If you ignore the decimal point and divide just as you would divide the integer 20 you see that 5 is the quotient digit. Where does the decimal point belong? Will the answer be 5 (5 ones)? No, it can't be, because one of the 4 equal parts of 2 can't be as much as 5; in fact, 2 ÷ 4 is less than one. Another way we can write 2 divided by 4 is $\frac{2}{4}$. You know that $\frac{2}{4} = \frac{1}{2}$ and that $\frac{1}{2} = .5$ (the decimal fraction). Good judgment helps you to see that the decimal point belongs at the left of 5; **.5** is our answer.

$$2.0 \div 4 = .5$$

You can check if you wish:

Mentally	or	On Paper
5 tenths is equivalent		.5
to one half. Four halves		$\times 4$
equal two wholes.		2.0

Did careful thinking make this example very simple?

Try the examples in SET C. Copy the quotients and place the decimal points by thinking where they belong. Ask yourself if your answers are reasonable. For instance, in doing Example **1a** you might think as follows in locating the point: "In **6.3 ÷ 3 = 21** I am dividing a number slightly larger than 6 into 3 equal parts. How large will each part be? It will be 2, with a little over. The quotient digits are **2 1**; the point belongs at the right of the 2, so **2.1** is the answer."

Check mentally, or on paper if it gives you more confidence. Then turn to page 28 at the back of this booklet.

SET C

	a	b	c
1.	6.3 ÷ 3 = 21	4.8 ÷ 4 = 12	9.6 ÷ 3 = 32
2.	4.0 ÷ 5 = 8	14.4 ÷ 9 = 16	68.8 ÷ 8 = 86
3.	57.5 ÷ 25 = 23	95.0 ÷ 10 = 95	60.0 ÷ 15 = 40
4.	82.2 ÷ 6 = 137	25.5 ÷ 5 = 51	135.8 ÷ 2 = 679

You are now ready to look at some examples which have more digits in them but which still require only one thing for you to be able to do them. What is it? That's correct, your use of good judgment.

Where does the decimal point belong in the quotient of the following example: **29.61 ÷ 63 = 47**? The integer in the dividend is 29, so we ask if 63 is contained in 29 forty-seven times. Is it contained once? Can either 47 or 4.7 be the quotient? What about .47?

You know that you can think of a division example as a fraction. For example, $2 \div 4 = \frac{2}{4} = \frac{1}{2}$. If you round 29.61 to 30 and 63 to 60 you can think of $\frac{30}{60}$, which reduces to $\frac{1}{2}$. Since .47 is close to .50, or $\frac{1}{2}$, it appears that placing the point at the left of the 4 is correct, and that **.47** is the quotient.

You can check if you wish:

Mentally	or	On Paper
47 hundredths is close to .50, or $\frac{1}{2}$. One half of 63 is about 30, which is close in value to 29.61, my original dividend.		.47 × 63 141 282 29.61

Where does the decimal point belong in the quotient of this example: **6.095 ÷ 23 = 265?** Your thinking might be as follows:

"First I look for the integer in the dividend. I find that 6 is this integer. Now, how many times is the divisor, 23, contained in 6? $\frac{6}{23} =$? Obviously the quotient will be less than 1, so I can rule out any answer which has an integer in it. Thus 265, 26.5, and 2.65 are not possible answers. Is .265 reasonable?

I will round the dividend (6.095) to 6 to estimate the answer more easily. 6 ÷ 23 can be thought of as $\frac{6}{23}$, and $\frac{6}{23}$ is about one fourth. (Actually $\frac{6}{24}$ is one fourth, but I am approximating.) One fourth is .25, or .250 when written as a decimal fraction, and .250 and .265 are close in value.

I conclude that the decimal point belongs at the left of the 2; the quotient is **.265,** and I am now ready to check."

Mentally	**or**	**On Paper**
The quotient .265 is close to one fourth (.250). If I call the divisor 24 and take one fourth of it, I get 6 as a dividend. This is close in value to 6.095, my original dividend.		.265 ×23 795 530 6.095

Do you see that when you use your good judgment the number of digits in either the divisor or the dividend doesn't make any difference to you in accurately placing the decimal point? Are you confident enough at this stage to go ahead on your own with this kind of example? If so, copy the following example and locate the decimal point without looking at the booklet:

7.98735 ÷ 5 = 159747

Did you think, "The integer in the dividend is 7; the numerals at the right of the 7 following the point do not change the value of the 7 itself. Therefore I estimate by thinking of 7 ÷ 5, and ask how many 5's there are in 7. There is one 5, with some remainder. The point must belong at the right of the 1; **1.59747** is the answer." See how easy it is to locate the decimal point when you use good judgment!

Sometimes we have a dividend which has a decimal fraction and no integer. How would you reason out placing the points in the quotients of this type?

a. $.54 \div 2 = 27$ Could you think, "One of 2 equal parts (or one half) of 54 hundredths would be a little more than 25 hundredths. Where does the point go? Yes, at the left of the 2; **.27** is the answer."

b. $.36 \div 4 = 9$ Will $\frac{1}{4}$ of 36 hundredths be 9 hundredths? The quotient digit is 9, so what must you do? Yes, annex a zero at the left of the 9 to have the 9 in hundredths' place. **.09**

c. $.4812 \div 12 = 401$ Will $\frac{1}{12}$ of a little more than 4 tenths be as much as 4 tenths? Could there be a 4 in tenths' place? No, you must annex a zero at the left. **.0401** is the answer.

Copy the quotients of SET D and locate the decimal points. Remember to consider the possibilities and then place the point with confidence because you know there is only one reasonable place to put it.

Check your work either mentally or on paper. Then turn to page 29 to check your thinking.

SET D

	a	b	c
1.	$15.3 \div 9 = 17$	$22.2 \div 6 = 37$	$8.96 \div 8 = 112$
2.	$6.824 \div 4 = 1706$	$37.50 \div 2 = 1875$	$3.5166 \div 6 = 5861$
3.	$839.6 \div 4 = 2099$	$39.41 \div 7 = 563$	$.96 \div 4 = 24$
4.	$.186 \div 3 = 62$	$.7280 \div 7 = 1040$	$.8163 \div 9 = 907$
5.	$40.29 \div 51 = 79$	$1178.1 \div 63 = 187$	$5.175 \div 45 = 115$
6.	$12.42 \div 3 = 414$	$39.71 \div 11 = 361$	$.0055 \div 5 = 11$
7.	$15.0 \div 30 = 5$	$2.701 \div 37 = 73$	$3.3 \div 4 = 825$

STAGE III

Decimal Fractions in Both Divisor and Dividend

If you have successfully used good judgment so far in locating the decimal point, STAGE III of your rocket is ready to fire. We are going to learn how to locate the decimal point in examples where there are decimal fractions in the divisor as well as in the dividend. Let's study some examples.

Step 1

Divide

$$2.1\overline{)6.3}\quad 3$$

63

1. The division has been done just as if $21\overline{)63}$ was the example.

Step 2

THINK—Locate the decimal point

$$2.1\overline{)6.3}\quad 3.$$

63

2. Now, think of $2.1\overline{)6.3}$. Where does the decimal point belong in the quotient? Is 2.1 almost 2? Is 6.3 almost 6? How many 2's are there in 6? Yes, 3, so the decimal point belongs at the right of the three: **3.**

Step 3

Check

$$\begin{array}{r} 2.1 \\ \times\ 3 \\ \hline 6.3 \end{array}$$

3. In a mental check we can round 2.1 to 2 and multiply. $3 \times 2 = 6$, which is close to 6.3, so we judge that the decimal point is correctly placed.

In a case like this, in which the division is even and the decimal point is at the end of the quotient, it is not always necessary to write the point. However, keep in mind that it is necessary to locate the point. You must know for sure where it belongs. It is a good idea to get into the habit of actually writing the point to be sure you really did locate it.

Step 1 Divide as if working with integers	$\begin{array}{r} 105 \\ 7.9\overline{)82.95} \\ 79 \\ \hline 39 \\ 00 \\ \hline 395 \\ 395 \\ \hline \end{array}$	1. Now look at this example. First the division was done just as if we were working with whole numbers: $79\overline{)8295}$. Now, where should the decimal point be placed in the quotient?
Step 2 THINK Locate the decimal point	$\begin{array}{r} 10.5 \\ 7.9\overline{)82.95} \\ 79 \\ \hline 39 \\ 00 \\ \hline 395 \\ 395 \\ \hline \end{array}$	2. Is 7.9 almost 8? Is 82.95 closer to 80 than 90? How many 8's are there in 80? Yes, ten. There will be about ten 8's in 82.95, so **10.5** must be the quotient. The decimal point is placed at the right of the 10.
Step 3 Check	$\begin{array}{r} 10.5 \\ \times 7.9 \\ \hline 945 \\ 735 \\ \hline 82.95 \end{array}$	3. A mental check would be to round 7.9 to 8 and 10.5 to 10. $8 \times 10 = 80$, which is almost 82.95, so we judge that the decimal point is in the right location.

What if you had decided that the answer was 105? Would there be as many as 105 eight's in 82.95? No. Suppose you had thought the answer was 1.05; would there be only about one 8 in 82.95? Again the answer is No. You can see that by thinking before you place the decimal point, you will discard any unreasonable answer.

Copy the quotients in SET E and locate the decimal points. Round numbers mentally to estimate a reasonable answer. Check your thinking on page 29.

SET E

	a	b	c	d
1.	$3.2\overline{)9.6}$	$5.2\overline{)20.8}$	$4.9\overline{)39.2}$	$2.1\overline{)48.3}$
2.	$5.3\overline{)15.37}$	$7.8\overline{)358.8}$	$26\overline{)29.64}$	$4.3\overline{)421.4}$
3.	$3.5\overline{)24.85}$	$2.8\overline{)10.472}$	$8.1\overline{)5.265}$	$49\overline{)254.8}$

Now you are ready to copy the quotients in SET F and to place the decimal point in each.

SET F

	a	b	c
1.	$2.1 \div 1 = 21$	$4.8 \div 2 = 24$	$15.5 \div 5 = 31$
2.	$9.6 \div 4.8 = 2$	$1.44 \div 7.2 = 2$	$29.61 \div 6.3 = 47$
3.	$19.2 \div 3 = 64$	$2.24 \div 2.8 = 8$	$28.64 \div 4 = 716$

Check your thinking by turning to page 29.

Here are two more sets of examples for you to do. Copy the quotients and locate the decimal points. Think carefully! Then check on page 30.

SET G

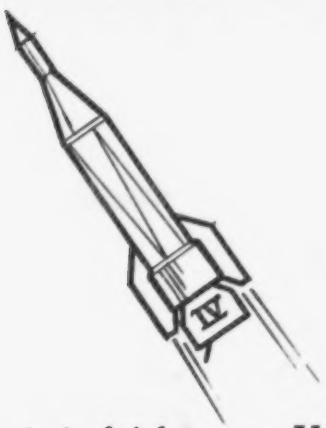
	a	b	c	d
1.	$4.8\overline{)27.36}$ (57)	$9.2\overline{)29.164}$ (317)	$3.7\overline{)3.256}$ (88)	$1.9\overline{)123.69}$ (651)
2.	$2.6\overline{)1.82}$ (7)	$32\overline{)14.08}$ (44)	$8.2\overline{)1.066}$ (13)	$3.4\overline{)2.924}$ (86)
3.	$2.8\overline{)1.204}$ (43)	$9.6\overline{).96}$ (1)	$21\overline{)153.09}$ (729)	$5.9\overline{)307.39}$ (521)

SET H

	a	b
1.	$91.8 \div 1.7 = 54$	$6.032 \div 5.8 = 104$
2.	$31.85 \div 1.3 = 245$	$1.312 \div 3.2 = 41$
3.	$14510.7 \div 2.3 = 6309$	$.9 \div 9 = 1$
4.	$170.24 \div 28 = 608$	$2.792 \div 4 = 698$
5.	$.37 \div 3.7 = 1$	$24.78 \div 7 = 354$
6.	$815.58 \div 1.8 = 4531$	$3.65 \div 3.65 = 1$

Since the next section is the last, it is vital. Through it you will reach the goal of real understanding in locating the decimal point in division with confidence and certainty. Your rocket will be secure in its orbit because you have placed it there through your own ability to reason and to use good judgment. You do not depend upon a device or gimmick which you are apt to forget, confuse, and never understand. You depend, instead, upon yourself.

STAGE IV
Keeping the Balance



Your rocket is ready to enter its fourth and final stage before going into orbit. All set? Here we go!

Perhaps you have been wondering about an example like this one: $.6\overline{)3.18}$. With a **decimal fraction** divisor, it is a little hard to judge how many times the divisor is contained in the dividend. But with a divisor that is an **integer,** it is fairly easy. Your problem, then, is to get a whole number in the divisor to help you in your thinking.

How can you get a whole number in the divisor: You may say, "Change the divisor." But if you change the divisor, won't the quotient change too? This question brings up the relationship of the dividend, divisor, and quotient. Let's think about the following set of examples.

a. $2\overline{)\$8.00}$ \$4.00

b. $4\overline{)\$8.00}$ \$2.00

c. $8\overline{)\$8.00}$ \$1.00

What is the dividend in **a**? in **b**? in **c**? It's the same, isn't it—\$8.00? Now look at the divisors. They increase from one example to the next—from 2 in **a** to 4 in **b** to 8 in **c**. What happens to the quotients when the dividend stays the same but the divisors change? The quotients change too, and in a definite way. We can state the relationship this way: **When the dividend is constant and the divisor increases, the quotient decreases.** (What do you think happens when the divisors decrease?)

If we studied many other such sets of examples we would find that when the dividend stays the same changing the divisor always changes the quotient.

If we can't change the divisor without changing the quotient, what can we do if we need to get a whole number in the divisor and still keep the quotient the same? Study the set of examples on the following page.

a. $2\overline{)\$8.00}$ → $\$4.00$

b. $4\overline{)\$16.00}$ → $\$4.00$

c. $8\overline{)\$32.00}$ → $\$4.00$

What is the dividend in each example? What is the divisor? Are either, or both, of them changed? Yes, each is doubled as you go from **a**, to **b**, to **c**.

Is the quotient for each example the same number?

The preceding set of examples illustrates an important principle: **It is possible to multiply (or divide) the dividend and the divisor of an example by the same number without changing the value of the quotient.** In examples **b** and **c** we compensated for doubling the divisor by doubling the dividend also. Some people say we "keep the balance." It is necessary to multiply (or divide) **both** dividend and divisor by the same number in order to keep the quotient the same.

Let's return to the question with which we started: How can you get an integer in the divisor of $3.18 \div .6$? You want a whole number in the divisor to help you use good judgment in locating the decimal point in the quotient.

Step 1

$.6\overline{)3.18}$ → $6\overline{)318}$

53
30
18
18

1. The division is completed as if whole numbers were being divided. The example is thought of as $6\overline{)318}$.

Step 2

$.6 \times 10 = 6.0$

$3.18 \times 10 = 31.80$

$.6\overline{)3.18}$ → 53

2. The decimal fraction .6 is changed to an integer. Multiplying by 10 will accomplish the change that is wanted. (You can ignore the zero at the right of the point.) Because you multiplied the divisor by 10, you must also multiply the dividend by the same number. Do you know why? Now carets (^) can be used to mark the new locations of the points in the new divisor and the new dividend.

Step 3

5.3

3. About how many 6's in 31? Yes, about 5, so where does the point go in the quotient? At the right of the 5; **5.3** is the answer.

You can now check:

Mentally	or	On Paper
Since 6 times a little more than 5 is about 30, 6 tenths times a little more than 5 would be about 3.		5.3 × .6 3.1 8

The method we just used is similar to the caret method described on page 9. The caret method works because of the dividend-divisor-quotient relationship, but it is applied by mechanically counting places with the numbers written out in the usual division form. In the new method you are now learning you will not need to mark changes in divisor and dividend by carets. Instead, you keep the changes in mind while you reason out placing the decimal point in the quotient.

Let's study another example, this time with a divisor of hundredths.

Step 1

```
      41
.24).984
     96
     24
     24
```

1. The division is completed as if whole numbers were being divided. $24\overline{)984}$ is the way we think of the example.

Step 2

```
  .24
× 10
2.40

 .984
× 10
9.840

      41
.2^4)9^84
```

2. It is easy to think, "How many 2's are contained in 9?" If you want to think of the 2 tenths (20 hundredths) as 2 ones, you must multiply the divisor by 10. (You can ignore the zero in the product.) If you multiply the divisor by 10, what must you do to the dividend? Yes, multiply it by 10 also.

   The carets show how you can now think of the divisor and dividend. How many 2's in 9? Yes, 4.

Step 3

```
     4.1
.2^4)9^84
```

3. Will the quotient be about 4? Where does the decimal point belong in the quotient? Yes, at the right of the 4; **4.1** is the quotient.

You may check:

Mentally	or	On Paper
.24 is almost $\frac{1}{4}$.		.24
One fourth × 4.1 is		× 4.1
about 1; .984 is about		24
1. The point is placed		96
correctly.		.984

Can you see that in order to reason out the place for the decimal point in the quotient you think of the first digit of the divisor (other than zero, of course) as if it were in ones' place? In the examples you have studied so far, you have learned to multiply the divisors by ten and to "keep the balance" by also multiplying the dividends by 10. Sometimes you will need to multiply by 100, 1000, and so on. These numbers are called **powers of ten.**

It is so easy to multiply by 10 and its powers that it really isn't necessary to show the work. What happens when you multiply a decimal fraction by 10? Study the examples below and see if you can figure this out.

	a	b	c	d	e	f
Multiplicand	.5	.9	.12	.48	.376	.759
	× 10	× 10	× 10	× 10	× 10	× 10
Product	5.0	9.0	1.20	4.80	3.760	7.590

Look at each multiplicand and each product. Do any of the **digits** change? No, except for the zero, the digit in each product is the same as that in its multiplicand. A 5 is still a 5, a 9 is still a 9, etc. What does change then? Yes, the value that each digit represents changes. Although the digit 5 is in both the multiplicand and the product of Example **a**, it is in **tenths'** place in the multiplicand and in **ones'** place in the product.

Now compare each product to each multiplicand. Does each digit in a product stand for ten times as much as it did in the multiplicand? The zeros at the right of the decimal point in the products here do not change the values of the numbers, so we can ignore them. We say that they are not **significant.**

Think about the multiplicand **.48** and the product **4.8** in Example **d**. In .48 the 4 is in tenths' place; in 4.8 it is in ones' place. The 8 has changed from hundredths' to tenths'

place. In .48 the decimal point is at the **left** of the 4, while in 4.8 the decimal point, which really serves to show us where the ones' place is, is at the **right** of the 4.

Since multiplying by 10 doesn't change the digits themselves but only their place values, the easy way to show this change in place value is to shift the decimal point one place to the right.

Try the method we have been discussing as you do the examples in SET I. The division is done for you; you merely copy the example, place the point, and check your answer.

Will you always need to change the place values? No, only when the divisor has no integer. Then you multiply both the divisor and the dividend by 10. Since this is your first practice, you will probably want to use carets to mark the changes. Of course, some of the examples may seem so simple to you that you will reason out the place for the point in the quotient without changing the divisor and the dividend, as in the example $.5\overline{)5}$. You may see immediately that .5 is contained in .5 just once and that **1.** is the quotient.

SET I

	a	b	c
1.	$.4\overline{).8}$ → 2	$.11\overline{).11}$ → 1	$.25\overline{)2.50}$ → 10
2.	$.7\overline{).5803}$ → 829	$.31\overline{)1.24}$ → 4	$.67\overline{)4.02}$ → 6
3.	$.8\overline{).64}$ → 8	$.7\overline{)41.65}$ → 595	$.61\overline{)27.45}$ → 45
4.	$7\overline{)24.64}$ → 352	$.164\overline{)3.772}$ → 23	$.101\overline{)1.1009}$ → 109
5.	$.9\overline{)4.887}$ → 543	$8.3\overline{)224.1}$ → 27	$.42\overline{)3.9732}$ → 946
6.	$3.2\overline{)6.72}$ → 21	$.9\overline{).3258}$ → 362	$.83\overline{).332}$ → 4
7.	$.315\overline{)1.3230}$ → 42	$28.5\overline{)108.585}$ → 381	$4.5\overline{)2.835}$ → 63

Check your thinking with the answers on page 30.

Now look at the example below.

a. $.02\overline{)8.64}$ See if you can figure out what you must do in order to get an integer in the divisor. Will multiplying by 10 work? $10 \times .02 = 0.2$ (or .2), so multiplying by 10 doesn't get the result which you want.

$$\begin{array}{r} .02 \\ \times\ 100 \\ \hline 02.00 \end{array}$$

Let's try multiplying by 100. Since the zeros do not change the value of the 2, you can ignore them.

$$\begin{array}{r} 8.64 \\ \times\ 100 \\ \hline 864.00 \end{array}$$

If you multiply the divisor by 100, you must "keep the balance" by also multiplying the dividend by the same number, as we do here.

b. $.02\overline{)8.64}$ (quotient: 432) The carets indicate that you now can think of the example as $2\overline{)864}$.

c. 432. The quotient digits are 432; now, where does the point belong? Yes, at the right of the 2. You can check by multiplying.

You have found that multiplying by 10 increases the place values of the digits in a numeral by moving them one place to the left. This change is shown by shifting the decimal point one place to the right. Now study the examples below and see what happens when you multiply by 10 and powers of 10.

a. $\begin{array}{r} .34 \\ \times\ 10 \\ \hline 3.40 \end{array}$ b. $\begin{array}{r} .34 \\ \times\ 100 \\ \hline 34.00 \end{array}$ c. $\begin{array}{r} .34 \\ \times\ 1000 \\ \hline 340.00 \end{array}$

What would be an easier way to do these multiplications? Compare **d** to **a**, **e** to **b**, and **f** to **c**.

d. .34 e. .34 f. .340

Does shifting the decimal points, as indicated by the arrows, accomplish the multiplications? How many places to the right is the point shifted to multiply by 10? by 100? by 1000? Notice that a zero is annexed at the right of the 4 in Example **f** in order to have enough places in the numeral.

Now do each of the examples in SET J by moving the decimal point to shift the place values (as in Examples **d**, **e**, and **f** above). Then check by multiplying. Compare your answers with those on page 31.

SET J

	a	b	c	d
1.	10×5.5	100×4.3	1000×2.17	$10{,}000 \times 8.63$
2.	$100 \times .06$	10×70	$10{,}000 \times .28$	1000×1.096

Just as a matter of interest: To **multiply** by 10 or powers of 10 you can change the place values by shifting the decimal point to the right. How, do you think, can you **divide** by 10 or its powers?

In SET K you will need to decide if you must multiply by 10 or a power of 10 in order to think of the first digit of the divisor, other than zero, as an integer. Also you will need to think of where you need to annex zeros. Study Examples **a**, **b**, and **c** below before trying SET K.

a. $.3\overline{)9}$ $\quad 3$

The division has been done as if we were dividing whole numbers. Does the quotient seem reasonable? A quick mental check will tell you that $3 \times .3$ is .9, not 9. The answer must be 30, mustn't it?

Or you might think in the following way: "If I think of the divisor as the whole number 3, then I must think of the 9 as 90." Do you have a mental picture like this? $3\overline{)90}$ $\quad 30$

30 is the quotient. Do you see that you had to think of annexing a zero in the dividend?

b. $.62\overline{)12.4}$ $\quad 2$

The division is completed as if with whole numbers. Where does the point belong? Think of the .62 and the 12.4 as if multiplied by 10. How many sixes are there in a little more than one hundred twenty? Do you need to annex a zero? Is 20 the quotient? $.62\overline{)12.4}$ $\quad 20$

c. $.05\overline{)2}$

Can you divide as if these were whole numbers? No, but what can you do? Yes, think of .05 as an integer by multiplying it by 100 and compensate by multiplying the 2 by 100. Can you see this in your mind? You needed to annex zeros in the dividend, didn't you? 40 is the quotient. $.05\overline{)2.00}$ $\quad 40$

Now copy SET K and work the examples. Check your thinking on page 31.

SET K

	a	b	c	d
1.	.7)7	.04).4	.35)3.5	.62).62
2.	.1)1	.44)13.2	.10)1	5.7)3.42
3.	.5)50	.12)10.8	3).3	.09)9

We have been thinking about annexing zeros at the **right** of numerals. Earlier you annexed zeros at the **left** of the first quotient digit. Let's think about an example of that type.

a. $\begin{array}{r} 2 \\ .2\overline{).04} \end{array}$ You multiply the divisor by 10 in order to work with an integer. Thus 2 becomes the divisor. When you multiply the dividend by 10 it becomes .04 .

b. $\begin{array}{r} 2 \\ 2\overline{).04} \end{array}$ What is half of four hundredths? Yes, two hundredths.

c. **.0**2 A zero is needed to make the quotient correct, that is, to show that the answer is two hundredths and not two tenths. This zero is annexed at the **left** of the quotient digit. It must be there if the answer is to be correct. You may check:

$$\begin{array}{r} .02 \\ \times\ 2 \\ \hline .04 \end{array}$$

As you do the examples in SET L remember that you can annex zeros at the left of the first quotient digit as well as at the right of the last quotient digit. Use the caret when you need to, annex any necessary zeros, and **think.** Check your work by turning to page 32.

SET L

	a	b	c	d
1.	.3).03	.2).2	3).09	20)2
2.	5).15	.8).08	16).16	159)15.9
3.	.7).0007	25)250	66).0132	15).90
4.	6).012	3.6)72	.83).00249	19).1083

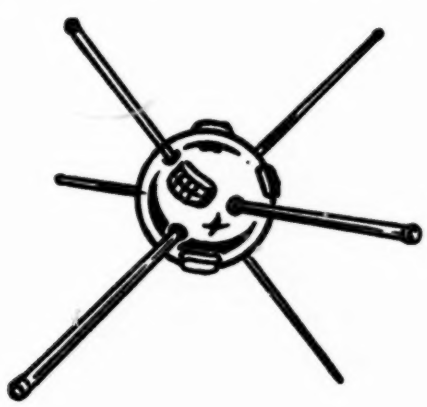
Now, imagine that you have a job which required that you use a calculating machine or a slide rule to perform the divisions shown in SET M. You have completed the divisions—now place the decimal points in the quotients by good judgment. Check your thinking on page 32. Your rocket is about to go into orbit!

SET M

	a	b	c
1.	$.6 \div 2 = 3$	$\$20.80 \div 5 = \$416$	$16.8 \div 4.2 = 4$
2.	$3 \div 5 = 6$	$8.184 \div 24 = 341$	$.888 \div .222 = 4$
3.	$.84 \div 12 = 7$	$\$10.92 \div 12 = \$91$	$\$2.40 \div 3 = \$80$
4.	$4 \div .25 = 16$	$7.4112 \div 6 = 12352$	$\$0.45 \div 9 = \$5$
5.	$34 \div .85 = 4$	$299.60 \div 20 = 1498$	$\$0.75 \div 5 = \$15$
6.	$.08 \div 8 = 1$	$13.12 \div 4.1 = 32$	$44.8 \div 7 = 64$
7.	$1 \div 25 = 4$	$8.3426 \div 10.1 = 826$	$901 \div .53 = 17$
8.	$12 \div .4 = 3$	$31.964 \div 1.22 = 262$	$158.4 \div 4.4 = 36$
9.	$.060 \div 15 = 4$	$80.37 \div 1.9 = 423$	$.099 \div .165 = 6$
10.	$.008 \div .4 = 2$	$.252 \div .004 = 63$	$2.46 \div .82 = 3$

How did you do? Do you believe that you were thinking clearly as you worked each example? Did you find that the caret helped? After a while you will probably be able to remember the changes and see where the decimal point belongs without using the caret. As you practice using good judgment you will find it easier and easier to locate the decimal point. You will be depending upon something which is always with you—your ability to reason—rather than merely memorizing a rule which you may later forget.

Congratulations! You are now in the orbit of thinking for yourself.



CHECKING YOUR THINKING

STAGE I

SET A

	a	b
1.	$4.32 ÷ 4 = $1.08	$12.60 ÷ 6 = $2.10
2.	$64.96 ÷ 2 = $32.48	$50.00 ÷ 5 = $10.00
3.	$145.60 ÷ 7 = $20.80	$345.36 ÷ 3 = $115.12

How well did you do? If you had more than one error, restudy pages 10 and 11 and try SET A again. If you made one error, do that example again, then continue with STAGE I. If you had all correct, congratulations! Continue with STAGE I.

SET B

	a	b
1.	$.48 ÷ 4 = $.12	$1.25 ÷ 5 = $.25
2.	$1.33 ÷ 7 = $.19	$.18 ÷ 3 = $.06
3.	$4.28 ÷ 4 = $1.07	$9.30 ÷ 10 = $.93
4.	$1152.09 ÷ 9 = $128.01	$1.65 ÷ 15 = $.11
5.	$6.00 ÷ 6 = $1.00	$30.00 ÷ 8 = $3.75
6.	$621.30 ÷ 3 = $207.10	$1.08 ÷ 12 = $.09

Did you remember to annex zeros at the left in Examples **2b** and **6b**? If you did not, restudy the explanation on page 11.

If you feel that you are using good judgment in locating the decimal points in division with dollars and cents in the dividend, you are ready to go on to STAGE II. Your rocket has successfully completed its first stage. If you feel that you are not ready to go on, restudy STAGE I.

STAGE II

SET C

	a	b	c
1.	6.3 ÷ 3 = 2.1	4.8 ÷ 4 = 1.2	9.6 ÷ 3 = 3.2
2.	4.0 ÷ 5 = .8	14.4 ÷ 9 = 1.6	68.8 ÷ 8 = 8.6
3.	57.5 ÷ 25 = 2.3	95.0 ÷ 10 = 9.5	60.0 ÷ 15 = 4.0
4.	82.2 ÷ 6 = 13.7	25.5 ÷ 5 = 5.1	135.8 ÷ 2 = 67.9

Did these examples cause you any trouble? If they did, restudy STAGE II up to SET C. Then try SET C again. If you did well with this exercise, continue.

SET D

	a	b	c
1.	$15.3 \div 9 = 1.7$	$22.2 \div 6 = 3.7$	$8.96 \div 8 = 1.12$
2.	$6.824 \div 4 = 1.706$	$37.50 \div 2 = 18.75$	$3.5166 \div 6 = .5861$
3.	$839.6 \div 4 = 209.9$	$39.41 \div 7 = 5.63$	$.96 \div 4 = .24$
4.	$.186 \div 3 = .062$	$.7280 \div 7 = .1040$	$.8163 \div 9 = .0907$
5.	$40.29 \div 51 = .79$	$1178.1 \div 63 = 18.7$	$5.175 \div 45 = .115$
6.	$12.42 \div 3 = 4.14$	$39.71 \div 11 = 3.61$	$.0055 \div 5 = .0011$
7.	$15.0 \div 30 = .5$	$2.701 \div 37 = .073$	$3.3 \div 4 = .825$

Did you do well or did you have trouble? If you did well, go on to STAGE III. If you had trouble, decide where you need to do some more studying in STAGES I and II. Then try SET D again.

STAGE III

SET E

	a	b	c	d
1.	$3.2\overline{)9.6}$ → 3.	$5.2\overline{)20.8}$ → 4.	$4.9\overline{)39.2}$ → 8.	$2.1\overline{)48.3}$ → 23.
2.	$5.3\overline{)15.37}$ → 2.9	$7.8\overline{)358.8}$ → 46.	$26\overline{)29.64}$ → 1.14	$4.3\overline{)421.4}$ → 98.
3.	$3.5\overline{)24.85}$ → 7.1	$2.8\overline{)10.472}$ → 3.74	$8.1\overline{)5.265}$ → .65	$49\overline{)254.8}$ → 5.2

If you had trouble, read the following paragraph. If you understand and made no errors, try SET F on page 18.

Examples **1a** and **1b** probably were easy. In **1c** did you approximate and think, "39.2 is close to 40. I am dividing by about 5. The answer should be almost 8." For **1d** you might have said, "I know that 2 and 3 are the quotient digits. There are certainly more than 2 two's in 48. The point belongs at the right of the 3." Now try SET E again.

SET F

	a	b	c
1.	$2.1 \div 1 = 2.1$	$4.8 \div 2 = 2.4$	$15.5 \div 5 = 3.1$
2.	$9.6 \div 4.8 = 2.$	$1.44 \div 7.2 = .2$	$29.61 \div 6.3 = 4.7$
3.	$19.2 \div 3 = 6.4$	$2.24 \div 2.8 = .8$	$28.64 \div 4 = 7.16$

In **2b** did you think, "One divided by 7. The answer will be less than one. The point goes at the left of the two."

If you had trouble, try SET F again. If not, continue.

SET G

	a	b	c	d
1.	$4.8\overline{)27.36}$ $\quad 5.7$	$9.2\overline{)29.164}$ $\quad 3.17$	$3.7\overline{)3.256}$ $\quad .88$	$1.9\overline{)123.69}$ $\quad 65.1$
2.	$2.6\overline{)1.82}$ $\quad .7$	$32\overline{)14.08}$ $\quad .44$	$8.2\overline{)1.066}$ $\quad .13$	$3.4\overline{)2.924}$ $\quad .86$
3.	$2.8\overline{)1.204}$ $\quad .43$	$9.6\overline{).96}$ $\quad .1$	$21\overline{)1530.9}$ $\quad 72.9$	$5.9\overline{)307.39}$ $\quad 52.1$

SET H

	a	b
1.	$91.8 \div 1.7 = 54.$	$6.032 \div 5.8 = 1.04$
2.	$31.85 \div 1.3 = 24.5$	$1.312 \div 3.2 = .41$
3.	$14510.7 \div 2.3 = 6309.$	$.9 \div 9 = .1$
4.	$170.24 \div 28 = 6.08$	$2.792 \div 4 = .698$
5.	$.37 \div 3.7 = .1$	$24.78 \div 7 = 3.54$
6.	$815.58 \div 1.8 = 453.1$	$3.65 \div 3.65 = 1.$

If you really understand, read the last paragraph of STAGE III and go on to STAGE IV. If not, study STAGE III again.

STAGE IV

SET I

	a	b	c
1.	$.4\overline{).8}$ $\quad 2.$	$.11\overline{).11}$ $\quad 1.$	$.25\overline{)2.50}$ $\quad 10.$
2.	$.7\overline{).5803}$ $\quad .829$	$.31\overline{)1.24}$ $\quad 4.$	$.67\overline{)4.02}$ $\quad 6.$
3.	$.8\overline{).64}$ $\quad .8$	$.7\overline{)41.65}$ $\quad 59.5$	$.61\overline{)27.45}$ $\quad 45.$
4.	$7\overline{)24.64}$ $\quad 3.52$	$.164\overline{)3.772}$ $\quad 23.$	$.101\overline{)1.1009}$ $\quad 10.9$
5.	$.9\overline{)4.887}$ $\quad 5.43$	$8.3\overline{)224.1}$ $\quad 27.$	$.42\overline{)3.9732}$ $\quad 9.46$
6.	$3.2\overline{)6.72}$ $\quad 2.1$	$.9\overline{).3258}$ $\quad .362$	$.83\overline{).332}$ $\quad .4$
7.	$.315\overline{)1.3230}$ $\quad 4.2$	$28.5\overline{)108.585}$ $\quad 3.81$	$4.5\overline{)2.835}$ $\quad .63$

SET J

	a	b	c	d
1.	55.	430.	2170.	86,300.
2.	6.	700.	2800.	1096.

If any example caused you difficulty, decide where your thinking is not clear. Skim this booklet to find the explanation which will help you. Then try another similar example which you make for yourself. If you can solve your problem for yourself you will not only get the real understanding that comes from thinking things out on your own, but you will also gain confidence in your ability.

Now continue with STAGE IV.

SET K

	a	b	c	d
1.	$.7\overline{)7.0}$ (10.)	$.04\overline{).40}$ (10.)	$.35\overline{)3.5}$ (10.)	$.62\overline{).62}$ (1.)

In 1a did you first divide as if you were dividing whole numbers? Then did you multiply both the divisor and the dividend by the same number to get an integer in the divisor? Why did you need to annex a zero in the dividend? How did you check to see if your answer made sense?

In 1c why did you not need to annex a zero in the dividend? How did you know you needed to annex a zero in the quotient?

2.	$.1\overline{)1.0}$ (10.)	$.44\overline{)13.2}$ (30.)	$.10\overline{)1.0}$ (10.)	$5.7\overline{)3.42}$ (.6)

Before you divided in 2c why did you need to multiply both the divisor and the dividend by 10?

3.	$.5\overline{)50.0}$ (100.)	$.12\overline{)10.8}$ (90.)	$3\overline{).3}$ (.1)	$.09\overline{)9.00}$ (100.)

Again, search out explanations for any trouble spots. If you had any difficulty with 1b, 2a, 3a, or 3d, go over the questions about 1a again.

If you need help with 2b, 2c, or 3b, review what was asked about 1c. Then continue with STAGE IV.

SET L

	a	b	c	d
1.	$.3\overline{).03}$ → .1	$.2\overline{).2}$ → 1.	$3\overline{).09}$ → .03	$20\overline{)2.0}$ → .1
2.	$5\overline{).15}$ → .03	$.8\overline{).08}$ → .1	$16\overline{).16}$ → .01	$159\overline{)15.9}$ → .1
3.	$.7\overline{).0007}$ → .001	$25\overline{)250}$ → 10.	$66\overline{).0132}$ → .0002	$15\overline{).90}$ → .06
4.	$6\overline{).012}$ → .002	$3.6\overline{)72}$ → 20.	$.83\overline{).00249}$ → .003	$19\overline{).1083}$ → .0057

In 1c did you first divide as if you were dividing whole numbers? Did good judgment help you know how many zeros to annex at the left in the quotient? Use the same good judgment in other examples of this kind.

In 4b did good judgment help you see that you needed to annex a zero at the right in the quotient?

SET M

	a	b	c
1.	$.6 \div 2 = .3$	$\$20.80 \div 5 = \$4.16$	$16.8 \div 4.2 = 4.$
2.	$3 \div 5 = .6$	$8.184 \div 24 = .341$	$.888 \div .222 = 4.$
3.	$.84 \div 12 = .07$	$\$10.92 \div 12 = \$.91$	$\$2.40 \div 3 = \$.80$
4.	$4 \div .25 = 16.$	$7.4112 \div 6 = 1.2352$	$\$0.45 \div 9 = \$.05$
5.	$34 \div .85 = 40.$	$299.60 \div 20 = 14.98$	$\$0.75 \div 5 = \$.15$
6.	$.08 \div 8 = .01$	$13.12 \div 4.1 = 3.2$	$44.8 \div 7 = 6.4$
7.	$1 \div 25 = .04$	$8.3426 \div 10.1 = .826$	$901 \div .53 = 1700.$
8.	$12 \div .4 = 30.$	$31.964 \div 1.22 = 26.2$	$158.4 \div 4.4 = 36.$
9.	$.060 \div 15 = .004$	$80.37 \div 1.9 = 42.3$	$.099 \div .165 = .6$
10.	$.008 \div .4 = .02$	$.252 \div .004 = 63.$	$2.46 \div .82 = 3.$

Look at the examples you missed. Did you miss any of these: **1b, 3b, 3c, 4c, 5c?** If you did, look at the examples again. Are they alike in any way? Can you figure out why you missed them? If you need help, turn back to STAGE I.

Now check to see if you missed any of these: **1a, 2a, 2b, 3a, 4b, 5b, 6a, 6c, 7a, 9a.** See STAGE II if you need help.

How about these: **1c, 6b, 7b, 8b, 8c, 9b?** See STAGE III.

And these: **2c, 4a, 5a, 7c, 8a, 9c, 10a, 10b, 10c?** See STAGE IV if you need help.

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